

Collisional Dark Matter and Scalar Phantoms

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Abstract

As has been previously proposed, a minimal modification of the standard $SU(3) \times SU(2) \times U(1)$ theory provides a viable dark matter candidate. Such a particle, a scalar gauge singlet, is naturally self-interacting—making it of particular interest given recent developments in astrophysics. We review this dark matter candidate, with reference to the parameter ranges currently under discussion.

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I. INTRODUCTION

Although the presence of dark matter in the universe appears to be no longer in doubt, the nature of the dark matter remains elusive. Spergel and Steinhardt [1] have suggested self-interacting dark matter as a possible solution to numerous discrepancies between predictions and observations in the standard dark matter scenario. Cold dark matter predicts overly dense cores in galaxies, and they argue that this can be resolved if the dark matter particles interact with each other with a substantial scattering cross section σ , while interacting only weakly with ordinary matter. The basic physics is easy enough to understand: collisional dark matter conducts heat, and as the core of a galaxy heats up it expands, thus lowering the central density. Furthermore, this form of dark matter may account for the discrepancy between theory and observation on small scale structure, the stability of galactic bars, and a number of other issues. [2] In particular, Ostriker [3] has shown that collisional dark matter can also account for the growth rate of massive black holes at the center of galaxies. For these scenarios to work it is estimated that σ/m , where m denotes the mass of the dark matter particle, must lie in the range $0.5\text{--}6\text{ cm}^2/\text{g}$, or equivalently, $2 \times 10^3\text{--}3 \times 10^4\text{ GeV}^{-3}$.

In this note we discuss a possible candidate within the standard $SU(3) \times SU(2) \times U(1)$ theory. In all likelihood, the standard theory descends from a theory at higher energy scales, a grand unified theory, or possibly a string theory. It seems plausible that in this descent there would appear numerous scalar fields X_a which happen to be singlets under $SU(3) \times SU(2) \times U(1)$. These fields would not couple to ordinary fermions, and would be unknown to the standard gauge bosons. In general they can couple to Higgs fields, and through the Higgs fields feebly to ordinary fermions. The scalar field sector of the theory might read

$$\frac{1}{2} \sum_a m_a^2 X_a^2 + \frac{1}{4} \sum_a \sum_b \eta_{ab} X_a^2 X_b^2 + \frac{1}{4} \sum_a \sum_j \rho_{aj} X_a^2 (\varphi_j^\dagger \varphi_j), \quad (1)$$

where φ_j denotes Higgs doublets, and η_{ab} and ρ_{aj} denote numerous coupling constants. The X_a 's interact with each other directly through self interaction (the η term), or indirectly through Higgs exchange (generated by the ρ term). Fifteen years ago, Silveira and one of us [4] proposed these scalar particles, X_a (called “scalar phantoms”), as candidates for the dark matter. We would like to re-examine this possibility in light of recent developments, including improvements in our knowledge of the Higgs sector.

II. SELF-INTERACTING DARK MATTER CANDIDATE

Clearly the general case, with the set of parameters m_a , η_{ab} and ρ_{aj} , allows us a great deal of freedom. Following Silveira and Zee, we go to the simplest case of just one X field and one Higgs doublet, so that the relevant part of the Lagrangian reads

$$\frac{1}{2}m^2X^2 + \frac{1}{4}\eta X^4 + \frac{1}{4}\rho X^2(\varphi^\dagger\varphi) - \frac{1}{2}\mu^2\varphi^\dagger\varphi + \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2. \quad (2)$$

We have imposed the discrete symmetry $X \rightarrow -X$, which guarantees stability of the X particle, as the X field is odd under this symmetry while all other fields in the universe are even. The coupling η controls the self-interaction of the X , which we will choose to be in the range needed for cosmology. As we will see, this simplest case may be barely viable.

After spontaneous symmetry breaking, φ acquires a vacuum expectation value v with $v^2 = \mu^2/\lambda$. Writing $|\varphi(x)| = v + H(x)$ as usual, we have $m_H^2 \sim \mu^2$ and $m_X^2 = m^2 + \frac{1}{2}\rho v^2$. Replacing one of the φ fields in the term $\rho X^2(\varphi^\dagger\varphi)$ by its vacuum expectation value, we induce a cubic coupling of the form $\sim \rho v H X^2$. Since the physical Higgs field H couples to quark and lepton fields, this cubic coupling leads to a coupling of X to quark and lepton fields. As X couples to ordinary matter only via the Higgs field, it naturally interacts feebly with ordinary matter and could easily have escaped detection. Indeed, at low momentum transfer X interacts with a quark or lepton via the exchange of the Higgs field, giving an interaction amplitude $\sim \rho v f / m_H^2 \sim \rho m_f / m_H^2$. As usual, f denotes generically the Yukawa coupling of H to a quark or lepton and $m_f \sim f v$ denotes the mass of the quark or lepton. This assumes that there is only one φ field; with multiple φ fields these estimates are all loosened. Since, on the scale of particle physics, the masses m_f of the quarks and leptons in the first generation are rather small, the interaction of X with ordinary matter is further suppressed by a factor $\sim m_f / m_X$. In other words, as X knows about ordinary matter only through the Higgs, which itself couples very weakly to the first generation of fermions, its interaction with ordinary matter is necessarily weak, thus making X a natural candidate for the dark matter particle. (This also means that in the early universe, when fermions of the second and third generations were present in abundance, X might have played a significant role.)

In its simplest version our model is governed by three unknown parameters: η , ρ , and m_X . In the absence of experimental input, Silveira and Zee [4] made the natural choice

that ρ be roughly equal to λ , the Higgs self coupling, but this choice is certainly open to modification.

In the model of eq. (2) the scattering amplitude for $X + X \rightarrow X + X$ receives two contributions: $\sim \eta$ from the direct quartic coupling, and $\sim (\rho v)^2/m_H^2 \sim (\rho^2 \mu^2/\lambda)/m_H^2 \sim \rho^2/\lambda$ from Higgs exchange. The scattering amplitude is given by the larger of η and ρ^2/λ . As we know nothing about η , a natural assumption is to take η and ρ^2/λ to be comparable, but again this is not required. If we take the scattering amplitude for $X + X \rightarrow X + X$ to be of order $\sim \rho^2/\lambda$, then at low momentum transfer the differential cross section is given by $\sigma(X + X \rightarrow X + X) \sim (\rho^2/\lambda)^2/m_X^2$. Inserting $\sigma/m_X \sim (\rho^2/\lambda)^2/m_X^3$ into the Spergel-Steinhardt bound ($2 \times 10^3 \text{ GeV}^{-3} \lesssim \sigma/m_X \lesssim 3 \times 10^4 \text{ GeV}^{-3}$) we obtain

$$m_X \sim 0.03\text{--}0.08 (\rho^2/\lambda)^{2/3} \text{ GeV}. \quad (3)$$

For example, for an X particle with mass comparable to the nucleon we would need $6\sqrt{\lambda} \lesssim \rho \lesssim 13\sqrt{\lambda}$. This parameter range is entirely reasonable by the standards of particle physics. If, on the other hand, η is larger than ρ^2/λ , then eq. (3) is to be replaced by

$$m_X \sim 0.03\text{--}0.08 \eta^{2/3} \text{ GeV}. \quad (4)$$

III. COUPLING CONSTANTS

As pointed out in the previous section, our proposed dark matter candidate in the simplest version is governed by three parameters: the self-coupling, η , the mass of the particle, m_X , and the coupling to φ , given by ρ . Let us now study the limits set on their values by cosmological considerations. The parameters η , ρ , and m_X govern the temperature T_f at which X particles freeze out. This in turn determines the number density at freeze out, and thus the current number density and mass density of X particles. By demanding that a fraction of critical density $\Omega_X \sim 0.1$ is due to X particles, we are able to estimate the mass of the particle.

Following Lee & Weinberg [5], we are to solve the evolution equation

$$\frac{dn}{dt} = -\frac{3\dot{R}}{R}n - \langle\sigma v\rangle n^2 + \langle\sigma v\rangle n_0^2, \quad (5)$$

where n denotes the number density of X as a function of time t , n_0 the equilibrium number density of X , R the scale size of the Universe, and $\langle\sigma v\rangle$ the thermally averaged cross section

for annihilation of two X particles into fermions $X + X \rightarrow f + \bar{f}$. The first term on the right-hand side describes the expansion of the Universe, the second term accounts for particle annihilation, and the last term represents particle production. We note that the contribution of the cosmological constant to the critical density (estimated to be $\Omega_\Lambda \approx 0.7$ at present) stays constant as we go back in time, while the matter density increases as $1/R^3$. At freeze out, therefore, the cosmological constant is negligible.

The rate equation can be rewritten as

$$\frac{df}{dx} = \left(\frac{45}{8\pi^3 N_f G} \right)^{1/2} m_X \langle \sigma v \rangle (f^2 - f_0^2), \quad (6)$$

where $x = T/m_X$, $f(x) = n/T^3$, $f_0(x) = n_0/T^3$, G is the gravitational constant, and N_f is the effective number of degrees of freedom. Based on the numerical fit from Lee and Weinberg we obtain a numerical approximation to the value of f at freeze-out:

$$f(x_f) \approx 10 \left[\left(\frac{45}{8\pi^3 N_f G} \right)^{1/2} m_X \langle \sigma v \rangle \right]^{-0.95}. \quad (7)$$

The number density of X particles today is given by $n_0 = f(x_f)T_0^3$, where $T_0 = 2.7$ K = 2.4×10^{-13} GeV is the current temperature. The present mass density in X is given by $\rho_X = m_X n_0$, and so the contribution to closure density is

$$\Omega_X = f(x_f)T_0^3 m_X / \bar{\rho}, \quad (8)$$

with $\bar{\rho}$ the present day critical density needed to close the Universe.

The freeze out temperature, $T_f = x_f m_X$, is given by

$$\frac{1}{\sqrt{x_f}} e^{1/x_f} = \left(\frac{45}{16\pi^6 N_f G} \right)^{1/2} m_X \langle \sigma v \rangle. \quad (9)$$

IV. NUMBERS

Even in the simple model of eq. (2) there is a great deal of freedom in setting values for the different parameters. For the sake of definiteness, we take the annihilation cross section $\langle \sigma v \rangle$ to be [4]

$$\langle \sigma v \rangle \approx \frac{3(\rho m_f)^2}{4\pi(4m_X^2 - m_H^2)^2}, \quad (10)$$

where m_H is the mass of the Higgs, ρ is a coupling constant (from the $X^2 \varphi^+ \varphi$ term in the Lagrangian), and m_f is the mass of the heaviest fermion lighter than X .

Approximating the exponent in eq. (7) as -1 , and plugging this into eq. (8), we find that the explicit mass dependence drops out, [6] and

$$\Omega_X \sim 10 \sqrt{\frac{8\pi^3 N_f G}{45}} \frac{T_0^3}{\bar{\rho}} \frac{1}{\langle \sigma v \rangle} \quad (11)$$

$$\sim \frac{10^{-10} \sqrt{N_f}}{h^2 \langle \sigma v \rangle} \text{ millibarn}, \quad (12)$$

with the Hubble constant given by $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$. Utilizing our expression for the cross section, the contribution of X to the closure density comes out to be

$$\Omega_X \sim 10^{-9} \frac{\sqrt{N_f}}{h^2} \frac{(4m_X^2 - m_H^2)^2}{(\rho m_f)^2} \text{ GeV}^{-2}. \quad (13)$$

We can assume that ρ takes a value of order 1. For simplicity, we take $\rho \sim \frac{1}{8} g^2 (m_H/m_W)^2$, [4] with g the weak coupling constant and $m_W = 80 \text{ GeV}$ the mass of the W . For the range of parameters of interest to us, the heaviest appropriate fermion will be the bottom quark, so $m_f = m_b = 4 \text{ GeV}$. Taking $h \sim 0.7$, $N_f \sim 10$, and demanding that $\Omega_X \sim 0.3$, gives a mass of

$$m_X \sim m_H/2. \quad (14)$$

Plugging these values into eq. (9), we find that the freezing temperature is given by $x_f = T_f/m \sim 0.04$ (which depends only weakly on m_X). The particle is thus non-relativistic when it freezes out, as has been assumed throughout.

It is clear that a parameter range can be found for ρ and m_X such that the X can make a significant contribution to the dark matter in the Universe. However, from eqs. (3) and (4), our mass range implies high values for the coupling constants, suggesting that our perturbative approach is breaking down, and that our calculations are only to be taken heuristically. The simple case considered here, consisting of a single X field and a single Higgs doublet, is only marginally viable. As remarked earlier, the general version of our model allows us a great deal of freedom, and it is likely that trivial generalizations of this simple case could lead to viable dark matter candidates.

Note added: In the process of writing this note we learned that similar ideas were discussed by M. C. Bento, O. Bertolami, R. Rosenfeld, and L. Teodoro (Phys. Rev. D **62**, 041302(R) (2000)), and also by C. P. Burgess, M. Pospelov, and T. Veldhuis (hep-ph/0011335).

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